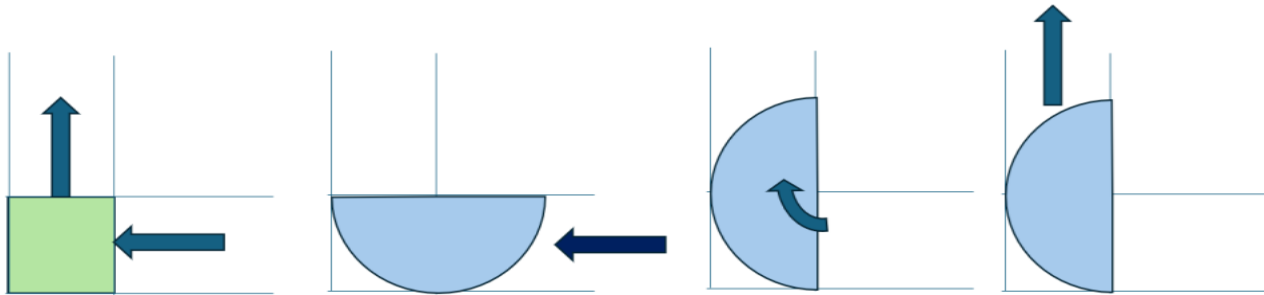


What is the largest sofa you can move through a corner?

Author and copyright: André Mariën

Introduction: the problem statement

The problem was stated a long time ago (1966, Leo Moser). You have a corner, 90°, same width both ways. What is the sofa with the largest surface you can slide through it? You cannot tilt or otherwise do operations on it: it is a 2-dimensional puzzle. Two simple examples show possible solutions:

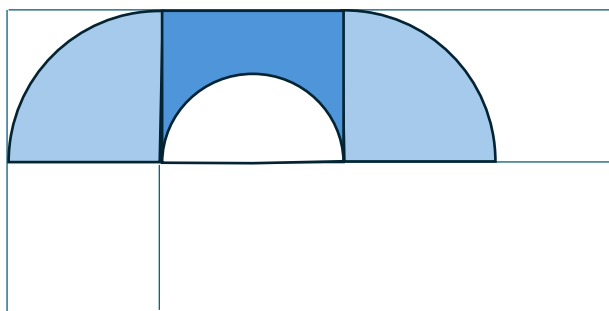


If the corridors are “1” wide, the first solution has a surface of “1”. The second solution has a surface of $\pi/2$. Can you do better?

When I recently heard about the problem it was said to be still unsolved. There are other such seemingly simple problems that are indeed still unsolved. I like to understand why they are indeed hard, not with any expectations to solve them.

The web page <https://www.quantamagazine.org/the-largest-sofa-you-can-move-around-a-corner-20250214/> is a good point to go to.

It starts indeed with the above solutions, and then a totally different one by John Hammersley, that is pretty good, the “telephone” (that old thing you held against your head).



- . two quarter-circles
- . A rectangle, with semi-circle cut-out

Is this the final and best solution? No! Its surface is about 2.2074 (precise: $\pi/2 + 2/\pi$). That is way better than the demi-circle result, adding the second term $2/\pi$.

The solution can be derived by logic, and proven, with one simple constraint, that may not be obvious (unless you know the solution can still be improved). This solution seems good enough for any practical cases I could think of, and generalizes as well (more on that later).

I knew a better one was claimed in 1992, Joseph Gerver of Rutgers University, with an area of approximately 2.2195. I didn't know someone claimed to have solved and proved that Gerver's solution was optimal when I started looking for the best solution.

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A fundamental approach for finding the best solution: challenge accepted

Initially the goal was to find the best solution, period. It could prove or disprove whether Gerver's solution was the best or not.

Till today, I did not look into how any of the previous very nice solutions were created, motivated or computed. The goal was an independent, motivated approach to finding the best solution, whatever it was.

While developing the solution I could reconstruct the solution of Hammersley, and some nice extension of this approach: any angle between the corridors. These solutions all have nice mathematical formulas.

Gerber's solution (and maybe: proof) remained the final challenge. In the end, I didn't bother.

The approach: long (indefinite) strokes of width “1”

The idea is positioning undetermined length strokes of width one, angle A , in all “reasonable” positions P over the corner, and repeat that for all angles. The position P can vary either vertically or horizontally or ... pick one: they are all ok if they cover all options.

The intersection $I(A,P)$ of the stroke with the corner in a certain position is the largest shape for the angle A and position P . Across all A , compute the total intersection of $I(A,P)$ for all A , P . Pick the one with the largest surface area. It is optimal.

It is also a huge computation task providing a numerical approximation only. Still it could be expected to prove or disprove solutions by providing upper and lower bounds for the size of the solution with a given precision.

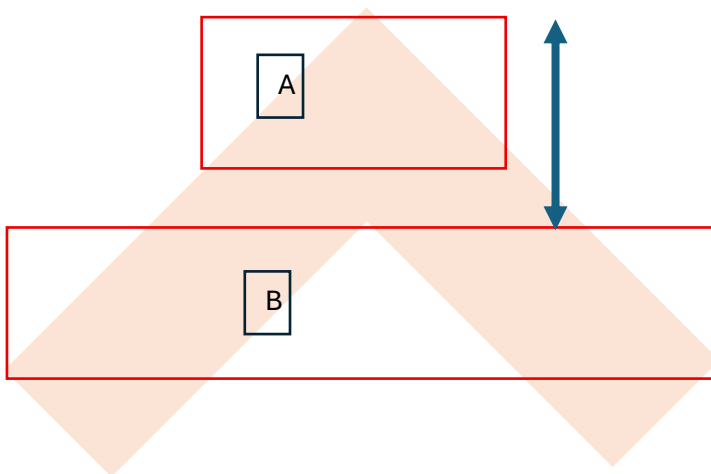
Making it practical

Some properties of the problem or solution can help reduce the brute force search operation.

Symmetry and 45°

The position of the stroke that makes an angle of 45° with the corner is the most restrictive position.

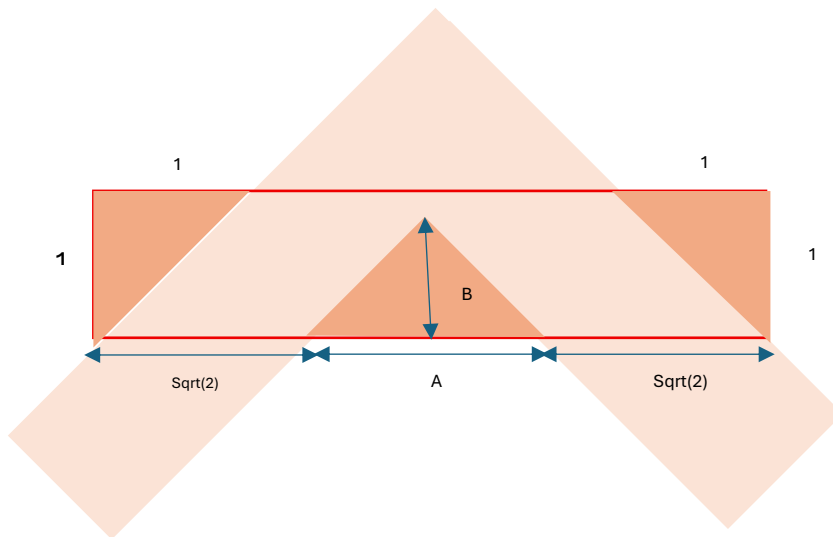
It makes sense to start from this position to reduce the stroke length. As the object must be in one piece the position can only vary within limited boundaries. The top one is the highest position, others are clearly smaller and provide no benefit. The bottom one is the edge to keep to one connected object.



Given the information that there is a solution with value 2.2074, any surface lower than this number is useless.

It is possible to determine the maximal surface based on this restriction. The largest size is **2.828**. This is not a tight upper bound. This value, found independently, coincides with the finding of Hammersley: $2\sqrt{2}$.

```
# opp = (2*sqrt(2)+a)*1-2*(1/2)-a*b/2
# b = a/2
# opp = (2*sqrt(2)+a)*1-2*(1/2)-a*(a/2)/2
# opp' = 1-(2/4)*a = 0
# a = 2
```



As that makes $b = 1$, the figure is the one “just” connected by a point (position B)

Centre cut-out

The strokes obviously are allowed to overlap with the corner and sides. It is the intersection that determines the shape in that position and angle.

Unsurprisingly, the best solutions have a rather deep cut, roughly $2/3$ of the stroke width. This could be expected as a loss of space on the inside gains in principle a bigger area on the outside.

Other angles and cuts extend the gap, but limiting how much extra is key for a good solution. The end result is always something not far from a circle, more so at the top (where the delta is probably hard to measure) than further down.

The final solution adds a twist near the bottom, more on that later.

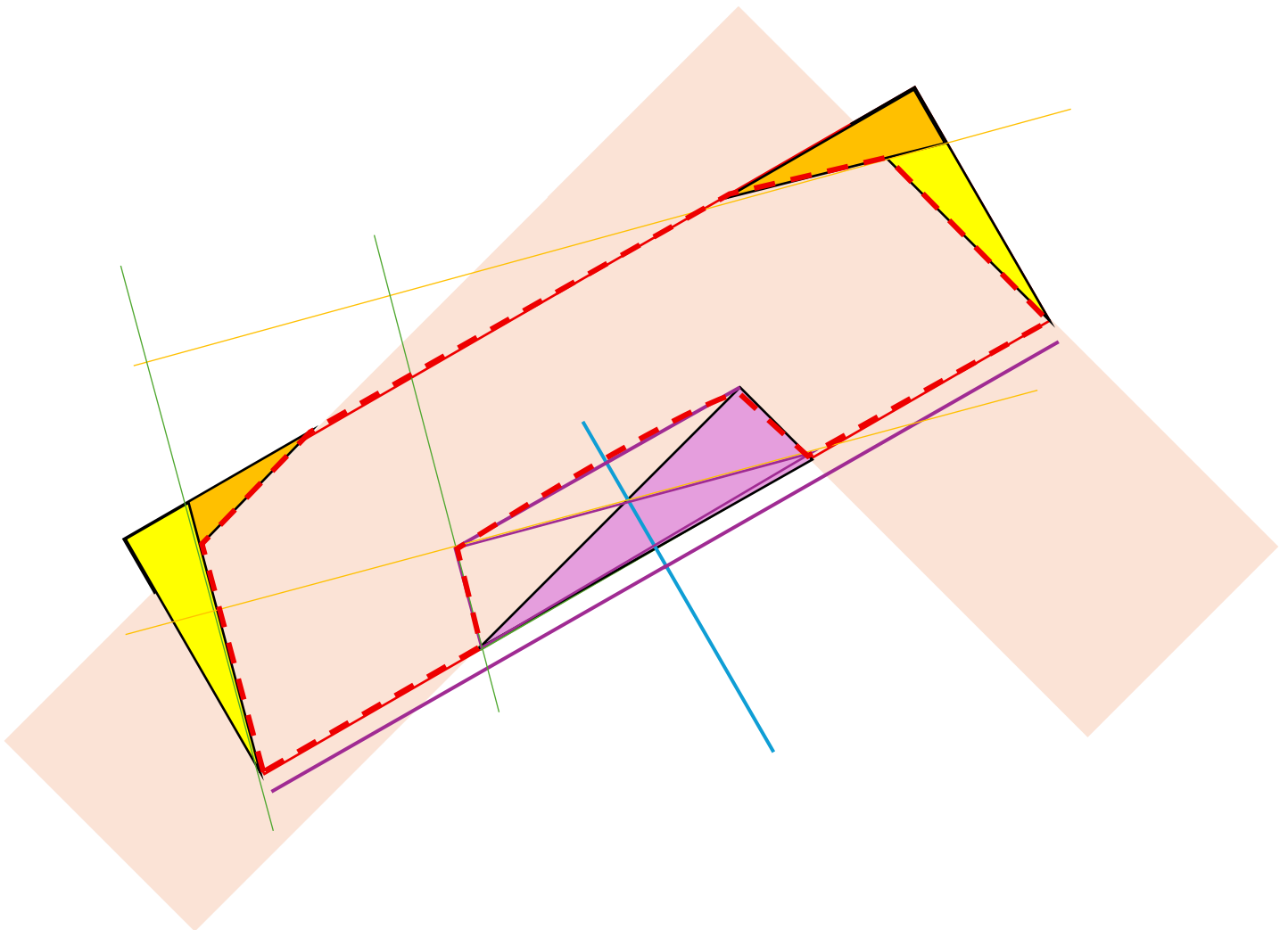
Using symmetry

The key to finding the solution faster is using the symmetry. The figure must make the same turn left as right from the middle position(45° with the corner). Pieces that drop of left, will drop of right symmetrically when making the turn to the right.

Let us consider the angel of 30 degrees. Its symmetrical turn is 60° . We also assume a symmetrical cut in the center of the figure.

We further assume that while turning, the gap on the top will be “filled” or rather cut out.

The following figure shows the resulting symmetrical object in red dashed lines.



Transformed to “standard” position, horizontally:

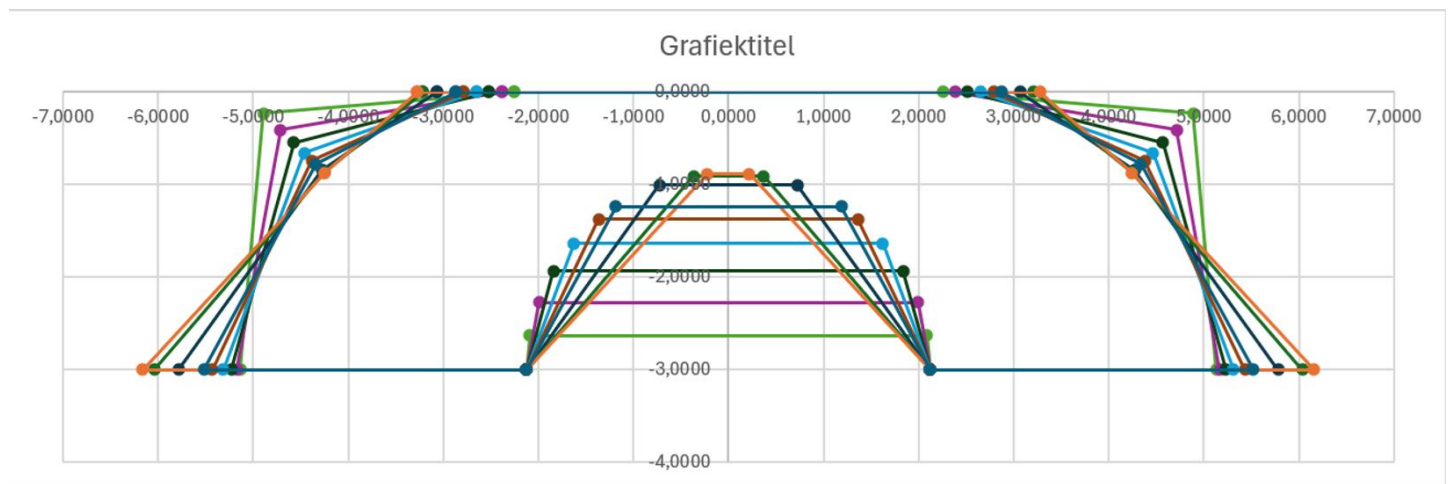
The figure has 10 corners, fully symmetric along the central vertical axis.

This figure can be used to produce an approximation of the solution of John Hammersley.

It seems to miss the capability to produce the more advanced solutions.



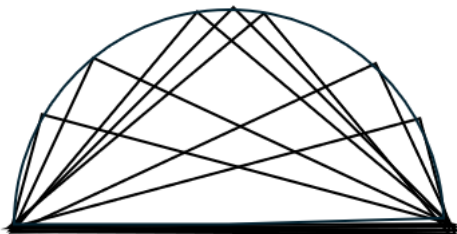
Hereafter you see what careful positioning of these figures for various angles achieves: the “telephone” appears naturally, even with limited angles it becomes clear.



It also hints very much in the direction of the Hammersley solution: one clearly sees the circles appear. Time for reflection: are these circles or near circles or ...?

Hammersley : central half-circle

The central gap is cut out by the corner of the corner. This are rectangular triangles with corners at the end of the gap:



As a consequence this figure is indeed a half-circle. The size of the circle depends on how deep the “cut” is. The surface gained by adding this central part is the area of the rectangle minus the area of the half-circle. There is an optimal size that can be computed.

$$\text{Rect} - \text{halfcirc} = 2 * r * 1 - \pi * r * r / 2$$

$$\text{Differentiate: } 2 - \pi * r = 0$$

$$R = 2 / \pi$$

$$\text{Total gap width} = 4 / \pi$$

Hammersley : quarter circles

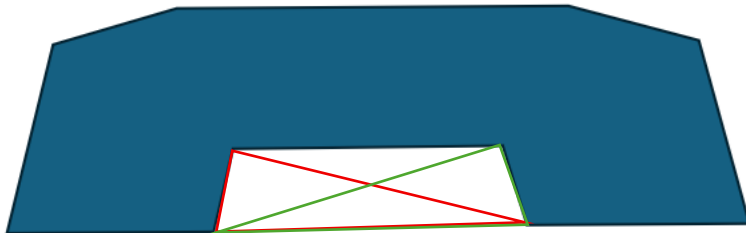
In mechanics they teach about movement composition. They split movements into components, typically a horizontal or vertical movement, combined with a circular movement. For the sofa problem we have a vertical, a horizontal and a circular movement. If we put an observer on a corner of the bottom central gap, he sees that side of the object making a perfect 90° turn around him, with the wall at the furthest side always touching the object at a distance that remains the same all the time: a circular movement. For the other side it is similar

Hammersley: final?

With the above considerations, the solution of Hammersley can be justified and computed correctly, using PI in the formula. At first glance, this must be the optimal solution too. It is not.

What remains to be investigated?

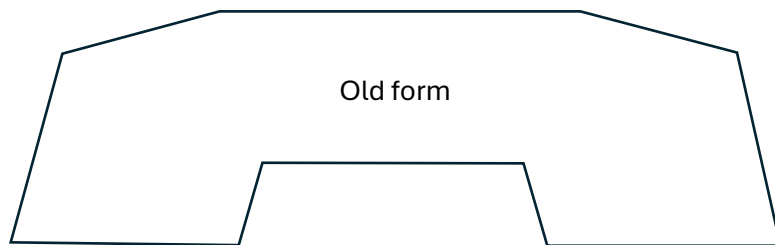
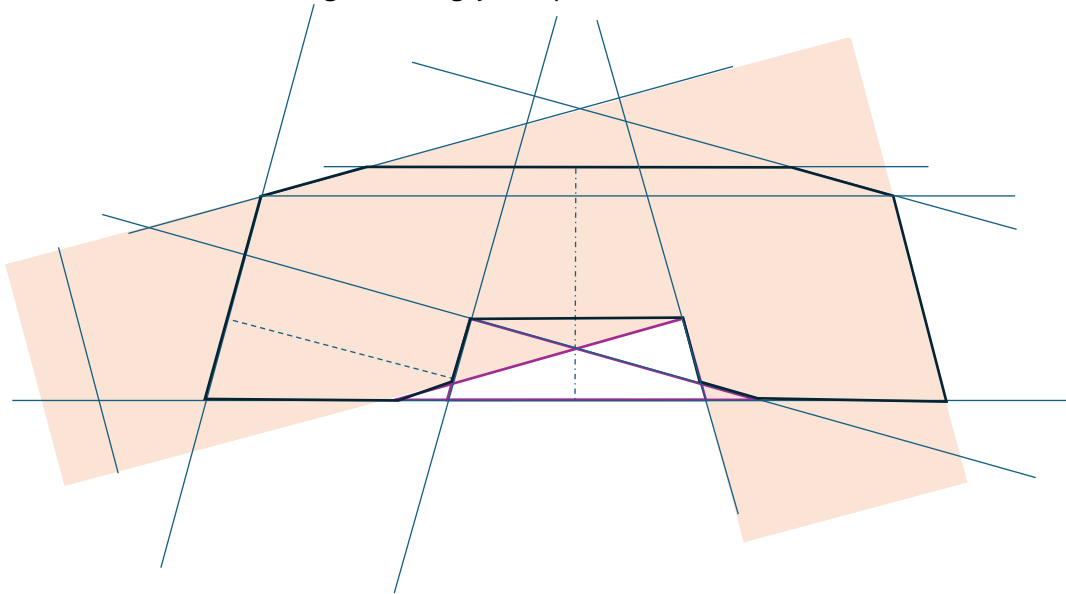
The method I proposed to use was supposed to guarantee the optimal solution. It seems like there must be a flaw if a better solution exists. The answer is in the central gap treatment.



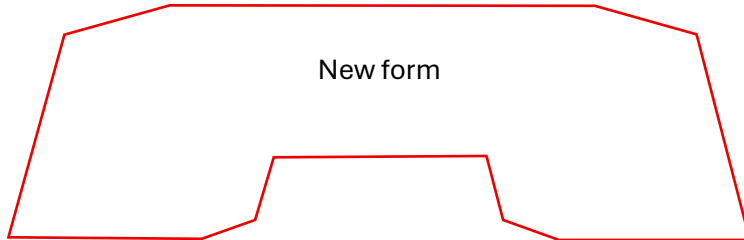
The central gap is the combination of the symmetric gaps turning left and turning right. They are aligned with their corners. THAT IS THE CLUE! There is no reason that **must** be the case. By deviating you sacrifice some space at the bottom and the center, but gain some at the edges. Will the gain be significant? No.

Central cut-out

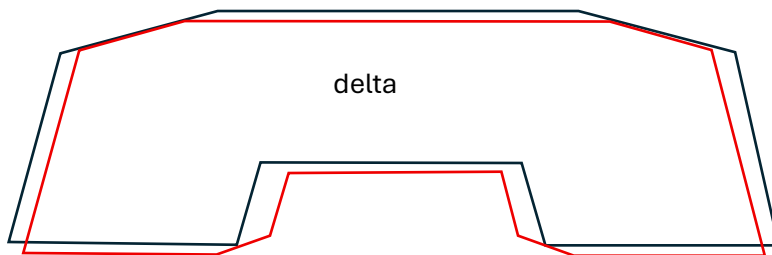
By varying the overlap in a different way for different angles instead of 100% overlap, one can create alternative solutions. There is no “magical” thing, just a parameter extra to use.



Old form

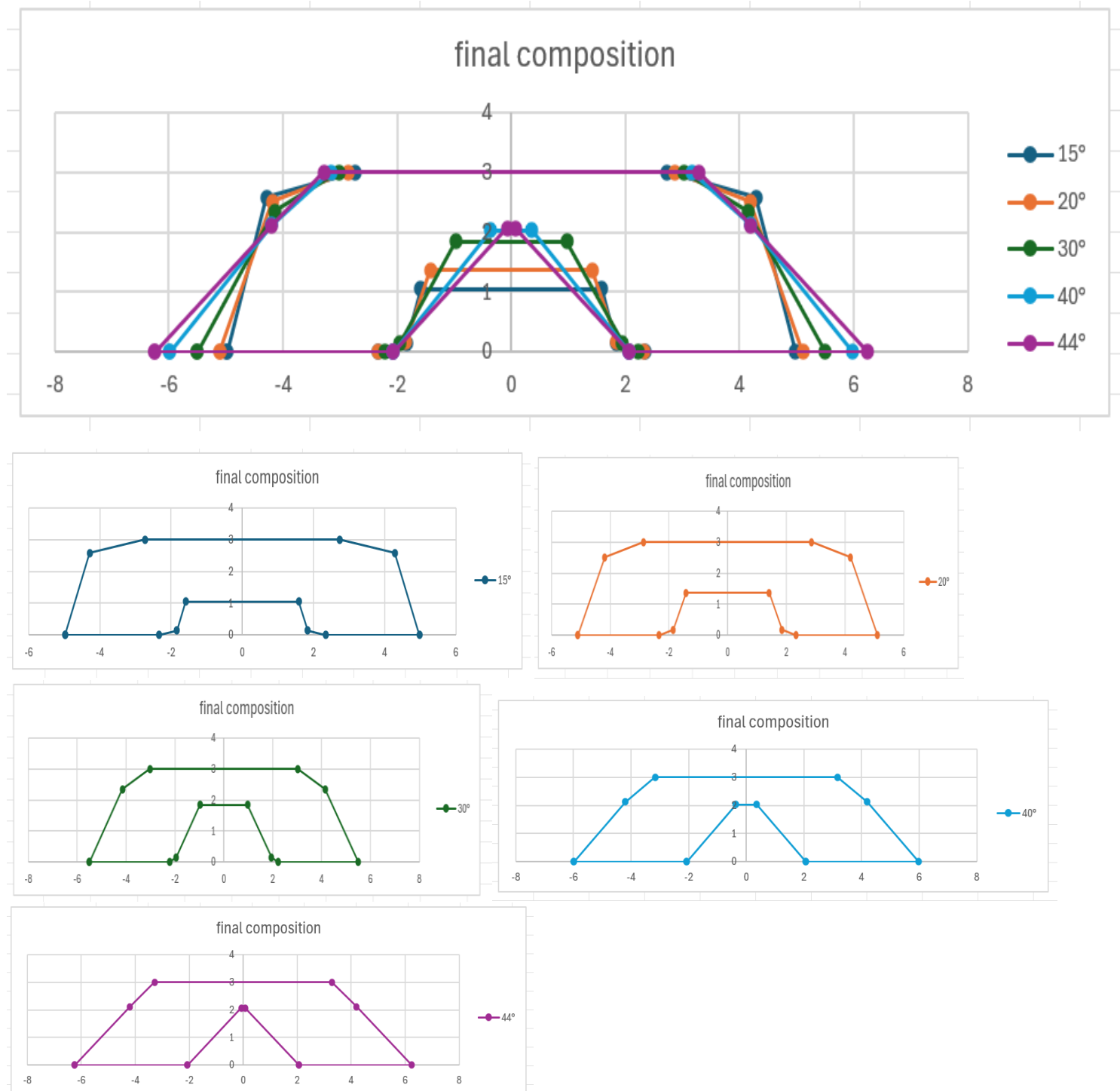


New form



delta

Below one can see how the “optimal” form may be reached



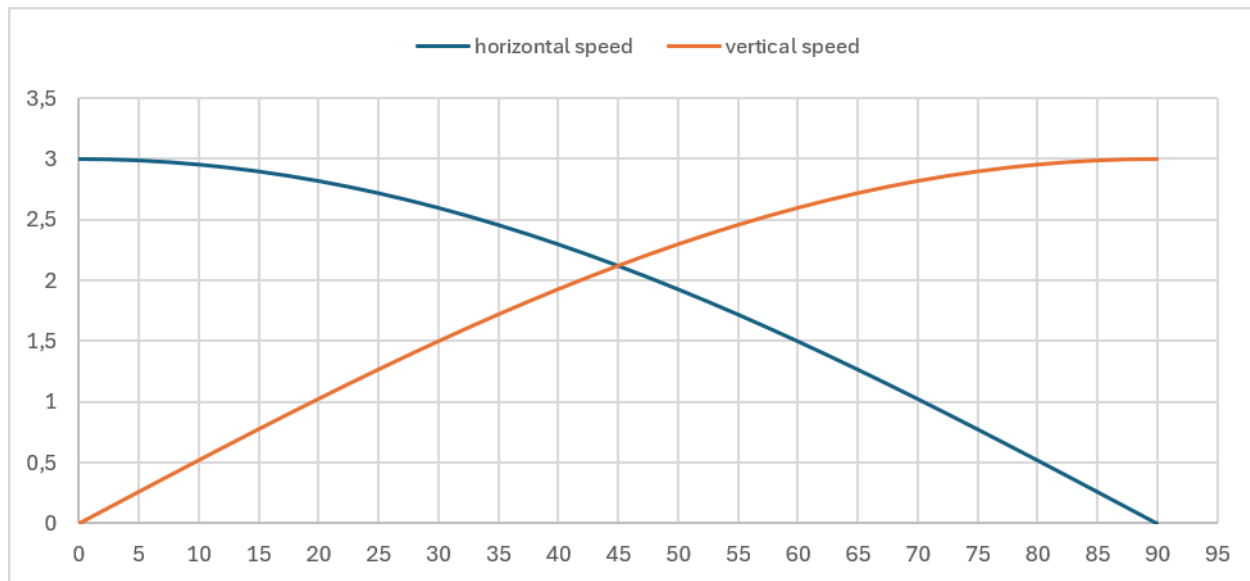
The results was obtained using “visual optimization”, keeping an eye on the left and right outer borders, avoiding “waste”.



Why would there be discrete changes? There should be continuous change

There is still an odd thing with the proposed ideal solution. The constructive method I use demonstrates that the whole solution can be build with gradual changes, there is no rupture anywhere. There are places where the delta between the solution of my approach and the proposed “best” are hardly measurable, if they exist. They would show up if computation were done with higher precision, I believe.

Fundamentally the object moves horizontally, vertically and turns. At no point is there a non-continuous change. I expect the solution to show this (when diving deep enough).

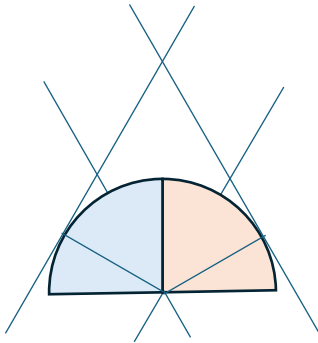


Extension: other corners

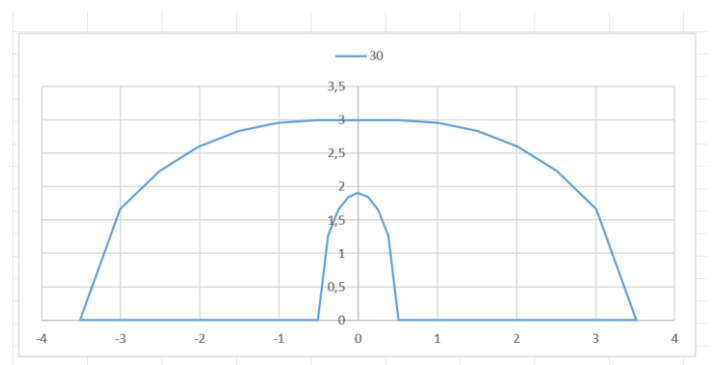
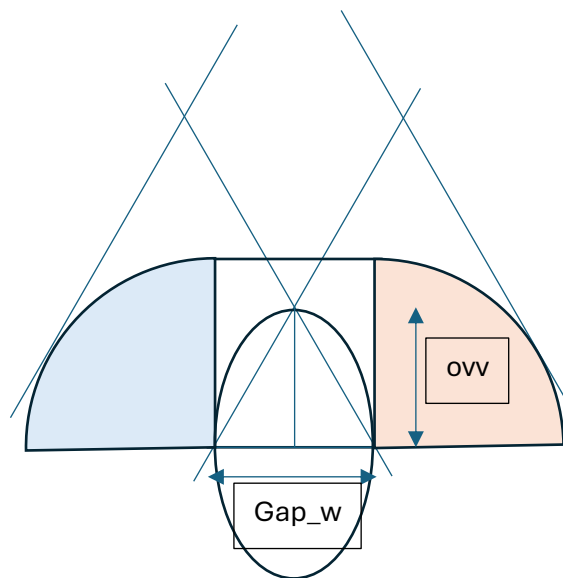
Sometimes a more general definition of the problem provides a nice solution for the more specific case. Or not. It would be nice to open up the problem and find the best solution for any angle.

Given the relatively small difference in surface between the Hammersley-style solution and more refined ones, I will go deeper in the Hammersley-style only: fixed gap, and both outer parts are semi-circles.

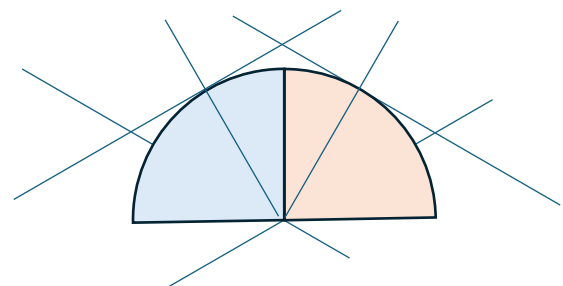
Corner angle 30°: basic solution



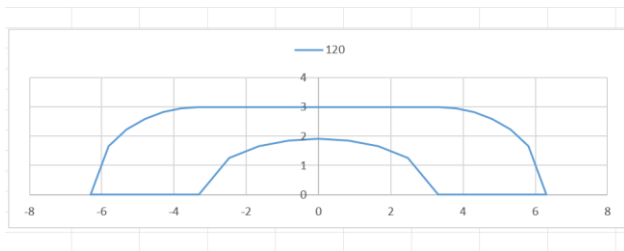
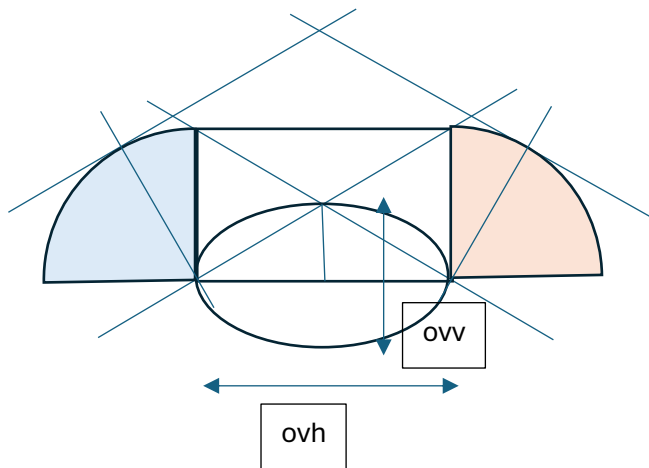
Corner angle 30°: idea: add central gap (oval shape ?!)



Corner angle 60°: basic solution (oval shape?!)



Angle 60°: add gap (oval shaped?!)



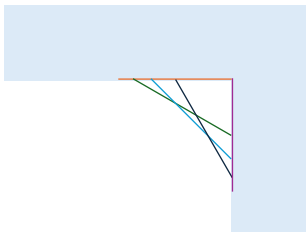
Unfortunately, the initial guess an oval would do the trick is **wrong**. The curves that you need are not that simple, while the combination of horizontal and vertical moves together with turning felt like that could be the case.

The connection of a point on the oval with the two extremes does not have the same value everywhere, whereas for a circle they do.

Turning around the angle using a fixed gap width

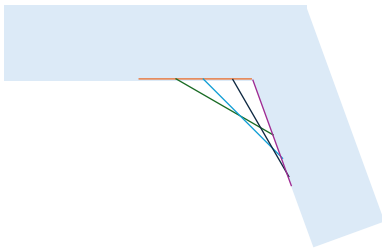
While this may seem simple looking at the 90° corner angle, for sharp angles it is far less obvious to see how the object would move around the corner.

90° corner angle: simple and symmetrical



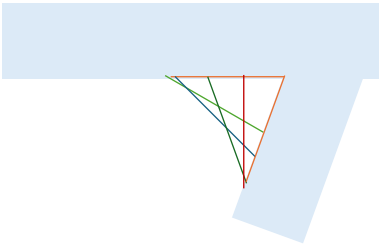
0° starting position
30°
45°
60°
90° end position

110° corner angle: simple



0° starting position
30°
45°
60°
70° end position

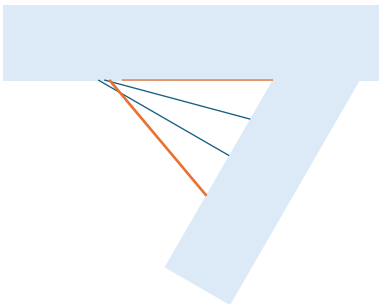
70° corner angle: simple?



This one looks simple.

However, If we make the gap larger, we cannot make this simple turn anymore. Similarly if we make the angle sharper, it looks like trouble!

Sharp corner angle 60°: complex

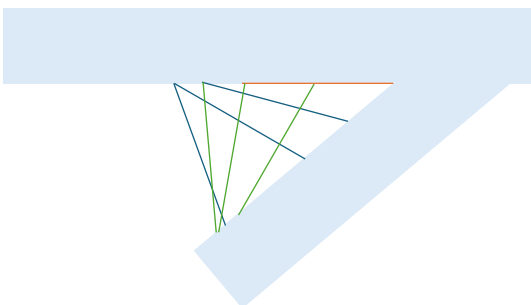


Observe how the initial movement of the left point is to the left!
But at some point the movement is back to the right.

The intuition is indeed that in order to make the turn, we need to go to the left till we have sufficient space to complete the turn clockwise.

That point is where we have both left and right corners with the line equal. Or when we reach the position where the gap line is orthogonal to the bisector of the corner angle. Or, it is the midway position of the turn.

Sharp corner angle 40°: sharper means more obvious complexity



start

1st part: moving left

2nd part: moving right

Why is the movement of the gap relevant?

The movement of the sofa has three components: a horizontal one, a vertical one, and a rotation. Once a gap is chosen, the three act as one: they are related.

The simplest case to demonstrate this is the 90° corner angle.



Except for the degenerate case (starting and ending positions), the gap width w is the hypotenuses of a triangle.

From the horizontal distance from the corner

- the vertical distance is known (Pythagoras using w)
- with both horizontal and vertical distance the tangents of the other corners is known

From the angle and the hypotenuses w we can determine both other sides of the triangle ($w \cdot \cos(\text{angle})$ for instance)

As the “sofa” is rigid, the movements of any point of the sofa are related to the movement of the gap corners.

Moving the Hammersley-style sofa through the corner

Computing and graphing: excel version

The core parameter for the Hammersley-style object is the gap width. For a clear distinction between an absolute “1” and the corner width we use $\text{corner width}=3$.

The second key parameter is the angle of the corner itself. We chose 0° as no corner at all, and 180° as a full change of direction.

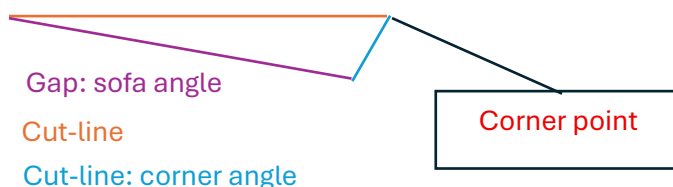
We compute the form placed horizontally, with the central gap point as $(0,0)$.

The key steps are:

- compute the gap width (gap_w)
- compute key points of the form (computations are simple):
 - the bottom far left and far right
 - the two points at the top of the central rectangle
 - the points at the end of the gap left and right
 - the central gap point
 - add some points of the circles left and right
 - add some points of the central semi oval
- rotate the form clockwise for the desired form angle around the left gap point $(-\text{gap_w}/2, 0)$
 - $rx = (x + \text{gap_w}/2) \cdot \text{sofa_a_cos} + y \cdot \text{sofa_a_sin}$
 - $ry = -(x + \text{gap_w}/2) \cdot \text{sofa_a_sin} + y \cdot \text{sofa_a_cos}$
- move the form horizontally so that the sharp point of the corner is always at the point $(0,0)$

To complete the picture, add the corner lines.

The horizontal move is less obvious to determine.



We draw a line from the right gap point with the corner angle. The intersection with the x-axis is our target point.

Point on the line: (RightGapx, RightGapy)

Direction: $\tan(\text{corner_angle})$

$\text{RightGapy} = \text{RightGapx} * \tan(\text{corner_angle}) + q$

$Q = \text{RightGapy} - \text{RightGapx} * \tan(\text{corner_angle})$

$\text{Corner_point_y} = \text{Corner_point_x} * \tan(\text{corner_angle}) + q$

$\text{Coner_point_y} = 0$

$\text{Corner_point_x} = -q / \tan(\text{corner_angle})$

Trouble: if the corner is 90° ...

Using the triangle sinus rule

We know 2 of the three angles, so we compute the third (purple/blue):

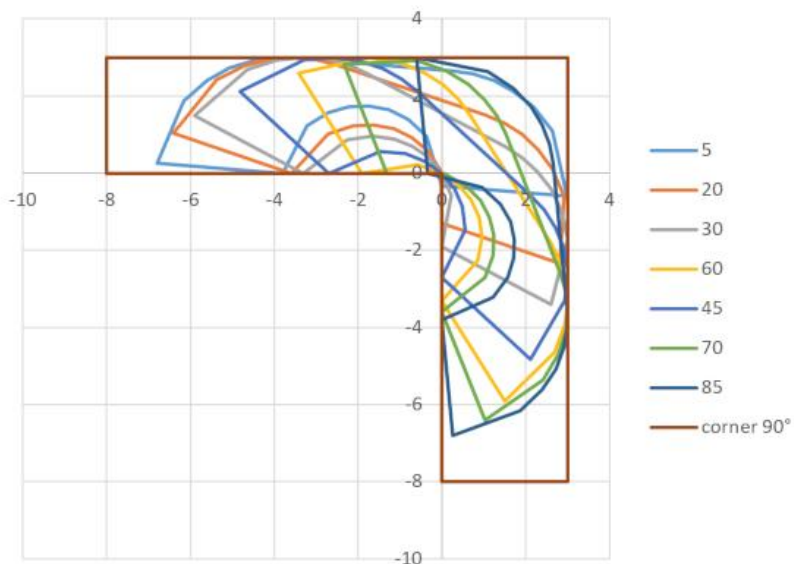
$\text{Angle} = \pi - (\text{sofa_a} + \text{crnr_a})$

$\text{Corner_point_x} / \sin(\text{angle}) = \text{gap_w} / \sin(\text{crnr_a})$

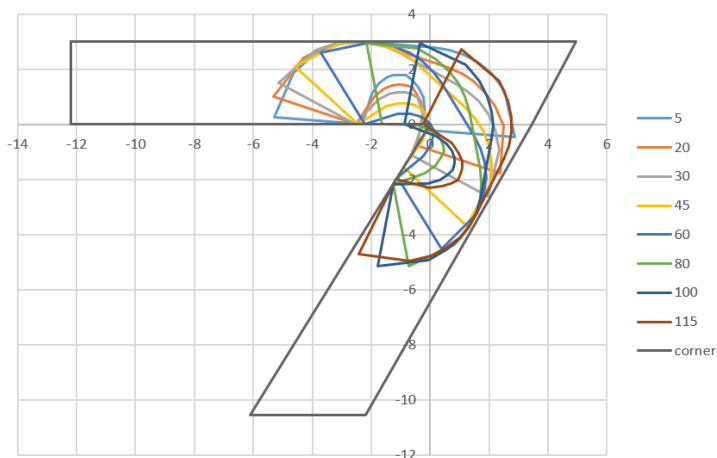
$\text{Corner_point_x} = \text{gap_w} * \sin(\text{angle}) / \sin(\text{crnr_a})$

Selected results

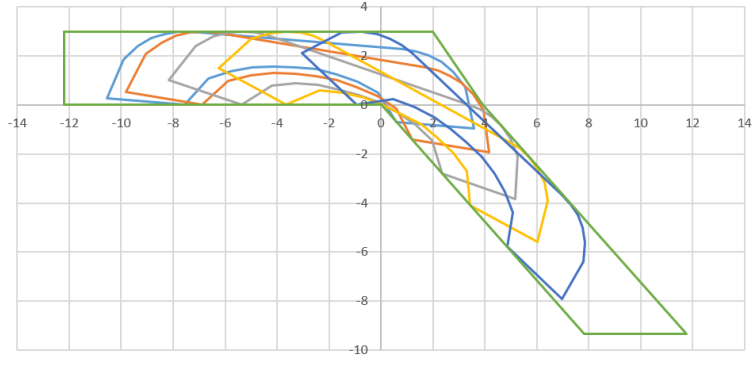
90° move



60°

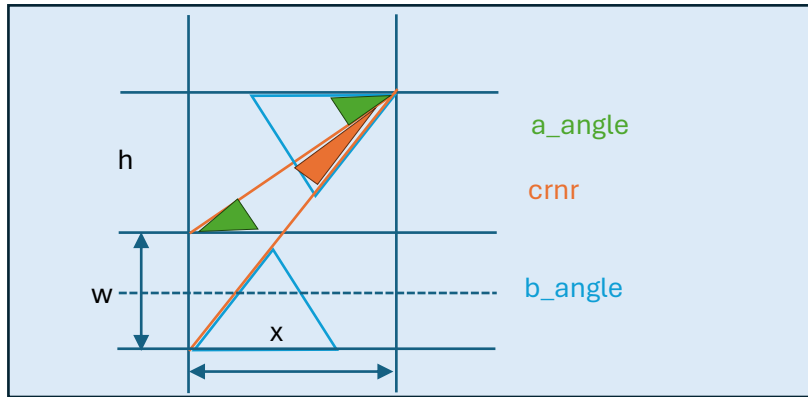


130°



How to compute the optimal fixed gap making the surface the largest possible (angles up to 90°)

Finding the formula for the curve



$$\begin{aligned}
 (h+w)/x &= \tan(b_angle) \\
 h/x + w/x &= \tan(b_angle) \\
 h/x &= \tan(a_angle) \\
 \tan(a_angle) + w/x &= \tan(b_angle) \\
 b_angle &= crnr + a_angle \\
 \tan(b_angle) &= \tan(crnr + a_angle) \\
 \tan(b_angle) &= [\tan(a_angle) + \tan(crnr)] / [1 - \tan(a_angle) \tan(crnr)] \\
 \tan(a_angle) + w/x &= [\tan(a_angle) + \tan(crnr)] / [1 - \tan(a_angle) \tan(crnr)] \\
 TA &= \tan(a_angle) \\
 TG &= \tan(crnr) \\
 WX &= w/x \\
 TA + WX &= (TA + TG) / (1 - TA * TG) \\
 [-TG] * TA * TA - [WX * TG] * TA + [WX - TG] &= 0 \quad \# \text{ final eq, ok} \\
 A &= -TG \\
 B &= -WX * TG \\
 C &= WX - TG \\
 D &= B^2 - 4 * A * C \\
 TA &= [-B \pm \sqrt{D}] / (2 * A)
 \end{aligned}$$

$$EQTA0: (-TG * WX + \sqrt{TG * (TG * WX^2 - 4 * TG + 4 * WX)}) / (2 * TG)$$

$$EQTA1: (-TG * WX - \sqrt{TG * (TG * WX^2 - 4 * TG + 4 * WX)}) / (2 * TG)$$

Use:

$$H = x * TA$$

$$Y = H + w/2$$

$$Y = x * TA + w/2$$

note: w/2 makes the solution symmetrical wrt the x-axis. It also ensures we will have a function.

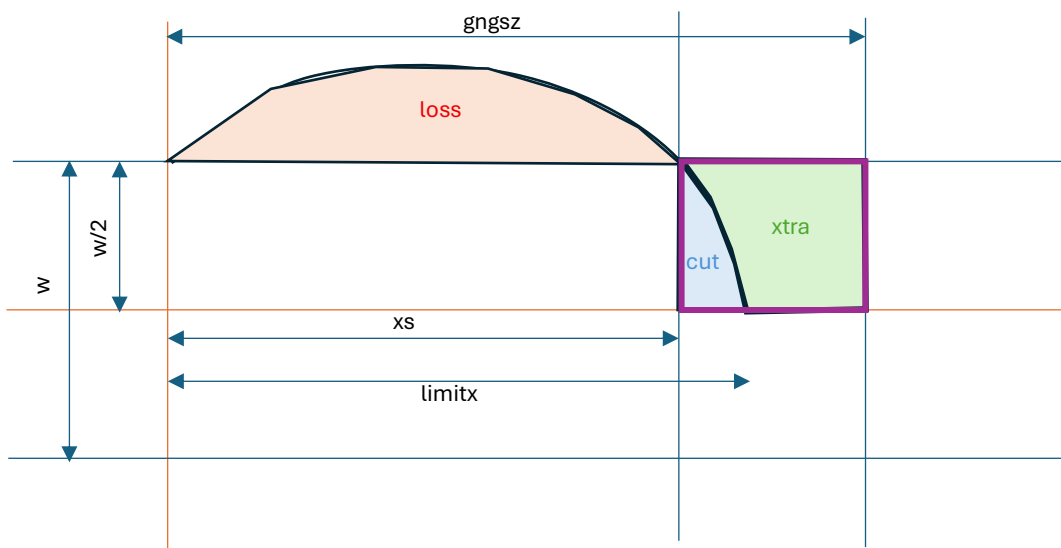
$$y = x * \sqrt{((w^2 * \tan(crnr) + 4 * x * (w - x * \tan(crnr))) * \tan(crnr) / x^2) / (2 * \tan(crnr))}$$

rewriting yields:

$$y = 1/2 * \sqrt{(w^2 + 4 * x * (w / \tan(crnr) - x))}$$

Note: verified using sympy for identical values (barring rounding errors)

Using the formula to compute the optimal fixed gap width for largest area



XS

```
y = sqrt(w**2 + 4*x*(w/tan(crnr)-x))/2
find xs, with f(xs) = w/2
```

$$w/2 = \sqrt{w^2 + 4 \cdot x_s \cdot (w/\tan(\text{crnr}) - x_s)} / 2$$

$$w = \sqrt{w^2 + 4 \cdot x_s \cdot (w / \tan(\text{crnr}) - x_s)}$$

$$w^*w = w^{*2} + 4 * xs^*(w/\tan(crn r) - xs)$$

$$4 \cdot x_s \cdot (w / \tan(\text{crnr}) - x_s) = 0$$

$$4 \cdot w / \tan(\text{crnr}) \cdot x_s - 4 \cdot x_s \cdot x_s = 0$$

$$w/\tan(\text{crnr}) - xs = 0$$

$$x_s = w / \tan(\text{crnr})$$

limitx

find limitx, with $f(\text{limitx}) = 0$

$$0 = \sqrt{w^2 + 4 \cdot \text{limitx} \cdot (w/\tan(\text{crnr}) - \text{limitx})}/2$$

$$(-4) \cdot \text{limitx}^2 + (4 \cdot w / \tan(\text{crnr})) \cdot \text{limitx} + (w^2) = 0$$

$$\text{sqrt}(d) = 4 * w * \text{sqrt}(1 + 1/\tan(\text{crnr})^2)$$

$$\text{limitx} = w/2 * (1/\sin(\text{crnr}) + 1 / \tan(\text{crnr}))$$

Finding “best” value for w

Compute based on 3 areas

Loss = $\text{integral}(f(x) - w/2, 0, x_s)$

Smallsq = $(g_{\text{ngsz}} - x_s) * w/2$

Cut = $\text{integral}(f(x), x_s, \text{limitx})$

Xtra = smallsq – cut

Netsrfc = xtra – loss

A sidestep to get around a BIG issue

The whole idea is to have a formula for the curve, allowing to compute the surface with an integral, and then differentiate for w, look for the zero value, and the corresponding w is the optimal one.

Unfortunately, the integral for our function is not a nice one: sympy does not find it.

Finding “best” value for surface for a given w: initial plan

Netsrfc' = (smallsq' – cut') – loss'

Nice idea, but as we cannot have the integral of f, f' is equally unknown: f is “close “ to F' but that is not good enough.

Using a numeric approach instead

Smallsq:

Smallsq = $(g_{\text{ngsz}} - x_s) * w/2$ # $x_s = w/\tan(\text{crnr})$
Smallsq = $(g_{\text{ngsz}} - w/\tan(\text{crnr})) * w/2$
Smallsq = $g_{\text{ngsz}} * w/2 - w * w / (2 * \tan(\text{crnr}))$

Cut:

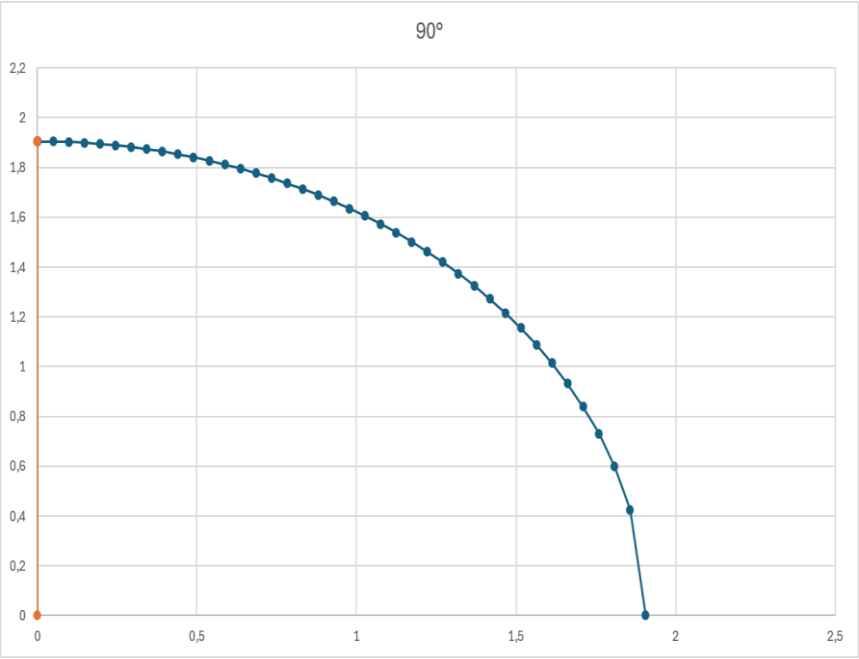
Cut = $\text{integral}(f(x), x_s, \text{limitx})$

Loss:

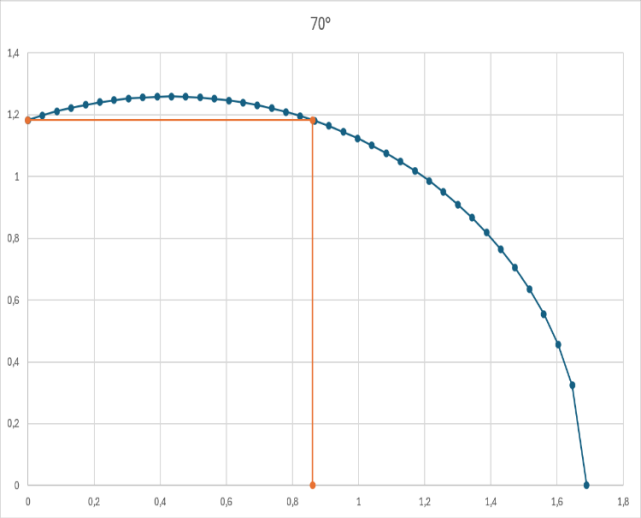
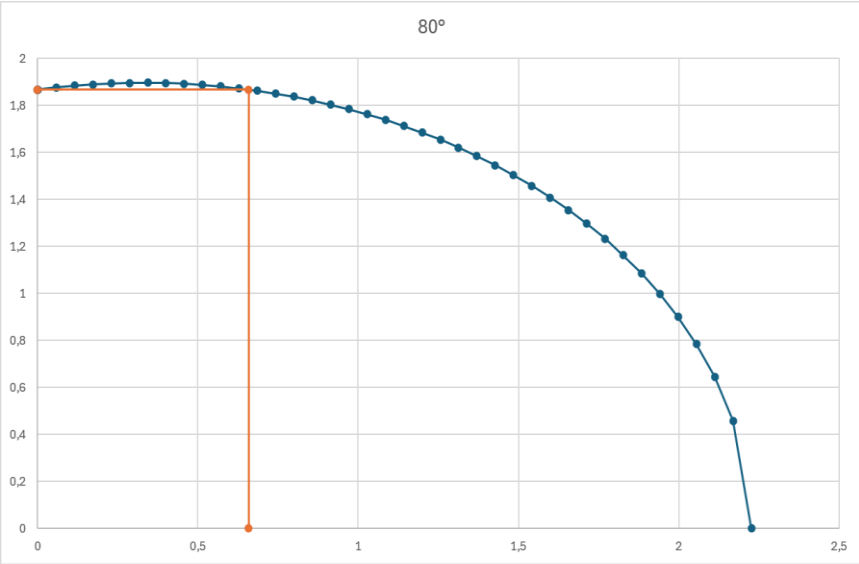
Loss = $\text{integral}(f(x) - w/2, 0.0000001, x_s)$

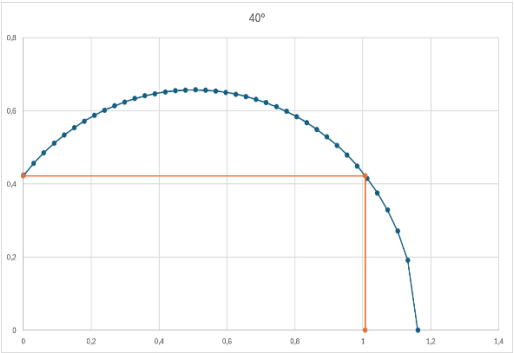
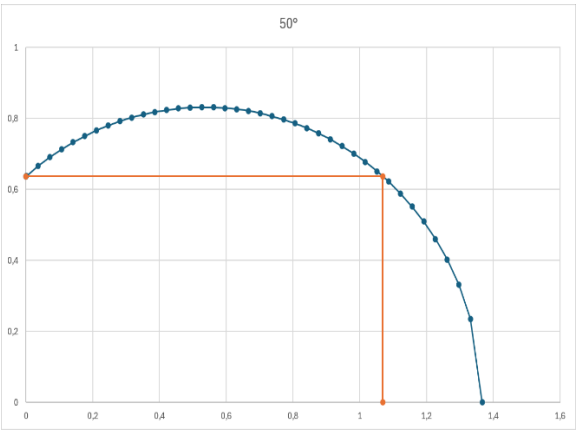
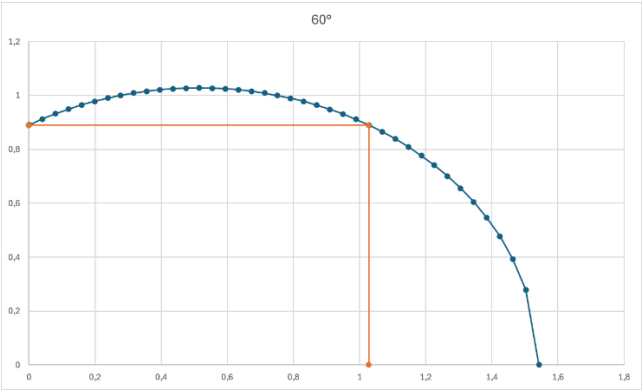
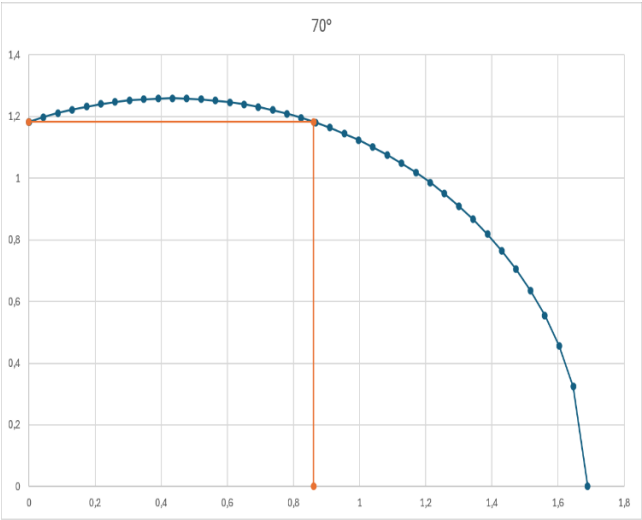
We can iteratively compute the gains by varying the “w” values, using for instance `sympy.Integral(f(x),(x,xs,limitx))`. This way we can find the optimal w (highest gain).

Individual graphs. Note: corner width = 3

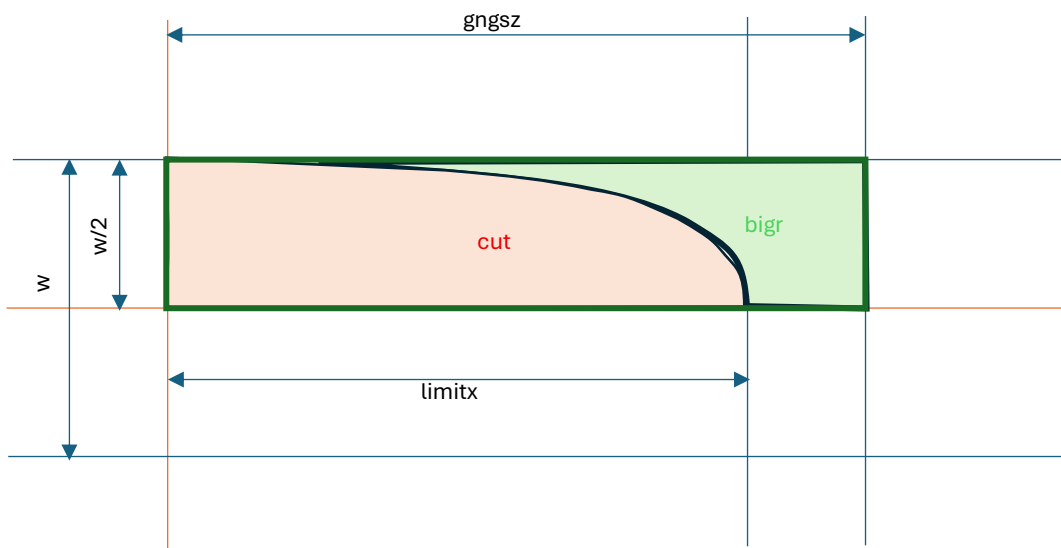


it's a circle!!





How to compute the optimal gap making the surface the largest possible (angles over 90°)



limitx

find limitx, with $f(\text{limitx}) = 0$

$$0 = \sqrt{w^2 + 4 \cdot \text{limitx} \cdot (w/\tan(\text{crnr}) - \text{limitx})} / 2$$

$$(-4) \cdot \text{limitx}^2 + (4 \cdot w/\tan(\text{crnr})) \cdot \text{limitx} + (w^2) = 0$$

$$\sqrt{d} = 4 \cdot w \cdot \sqrt{1 + 1/\tan(\text{crnr})^2}$$

$$\text{limitx} = w/2 \cdot (1/\sin(\text{crnr}) + 1/\tan(\text{crnr}))$$

Finding “best” value for w

Compute based on 2 areas

$$\text{Cut} = \text{integral}(f(x) - w/2, 0, \text{limitx})$$

$$\text{Bigr} = \text{gngsz} \cdot w/2$$

$$\text{Netsrfc} = \text{Bigr} - \text{cut}$$

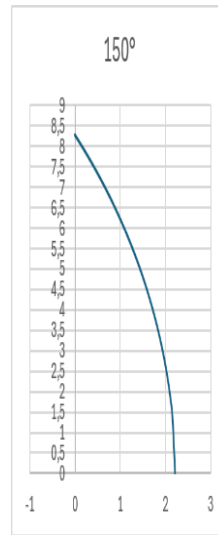
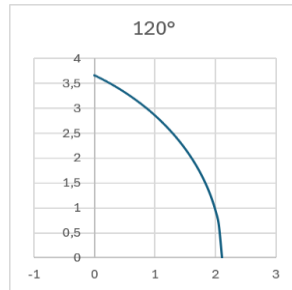
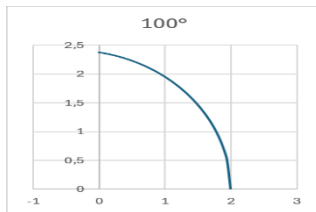
Finding “best” value for w for a given w

$$\text{Cut} = \text{integral}(f(x), 0, \text{limitx})$$

$$\text{Cut} = \text{integral}(\sqrt{w^2 + 4 \cdot x \cdot (w/\tan(\text{crnr}) - x)} / 2, 0.0000001, \text{limitx})$$

$$\text{With limitx} = w/2 \cdot (1/\sin(\text{crnr}) + 1/\tan(\text{crnr}))$$

Some graphs $> 90^\circ$



Remark

Notice how the above solution only focuses on the gap. To move from this Hammersley-style to Gerber-perfect (if it is – I admit not even trying to understand the mathematical proof) is an optimization and then we again have to consider gap differences in overlay and size.

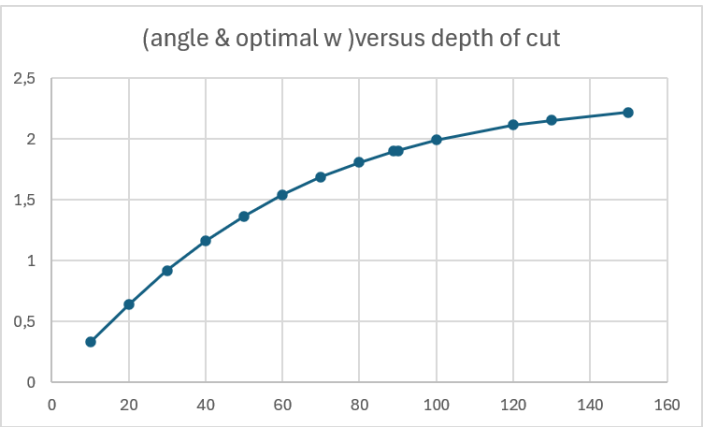
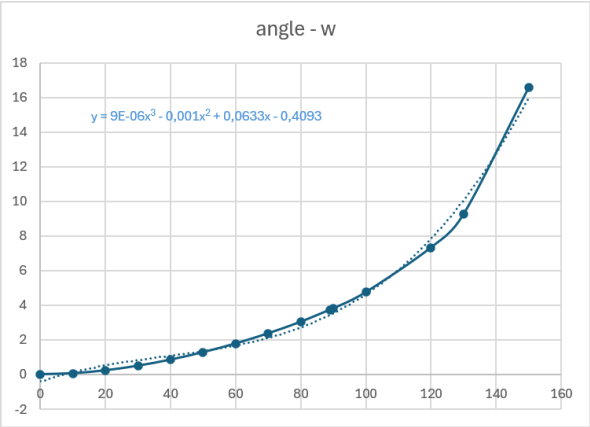
Results: overview table angles, gang width = 3x3

The following three tables provide the core results for angles 0 to 150.

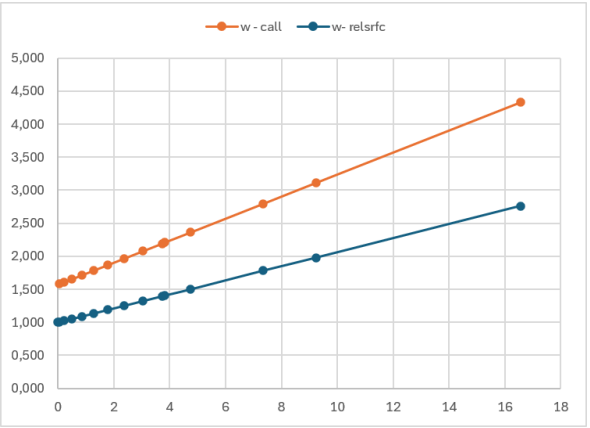
Corner angle	Gap width	Surface sofa	Calibrated to 1x1	Ratio of improvement over just 2 quarter circles
0	0	0	1,5	1
10	0,058	0,0865	1,580	1,01
20	0,2250	0,3381	1,608	1,02
30	0,4924	0,7375	1,653	1,05
40	0,8450	1,2666	1,712	1,09
50	1,2740	1,9126	1,783	1,14
60	1,7800	2,6707	1,868	1,19
70	2,3650	3,5459	1,965	1,25
80	3,0370	4,5548	2,077	1,32
89	3,7350	5,6033	2,193	1,40
90	3,81	5,7295	2,207	1,41
100	4,75	7,1239	2,362	1,50
120	7,33	10,9902	2,792	1,78
130	9.25	13,8889	3,114	1.98
150	16,565	24,8382	4,331	2,76

That blue row is the (identical) solution for 90° corner of Hammersley. The gap is indeed $4/\pi() \cdot 3$, as computed earlier.

Some graphs of relationships:



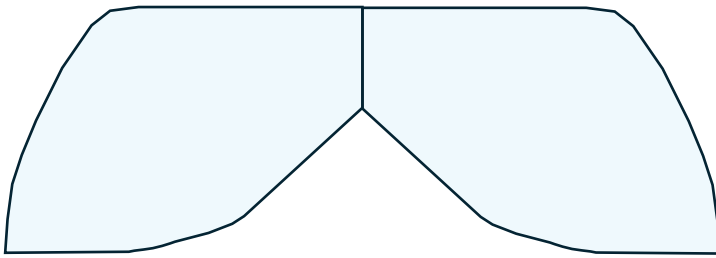
The bigger the angle, the larger the gap (increasingly), the deeper the cut (flattening).



Notice the linear correspondence between gap size and resulting size.

A side-step: an alternative form with non-trivial size: the “curtain”

While looking for possible solutions, an unexpected form showed up. It's surface is close to the semi-circle solution.



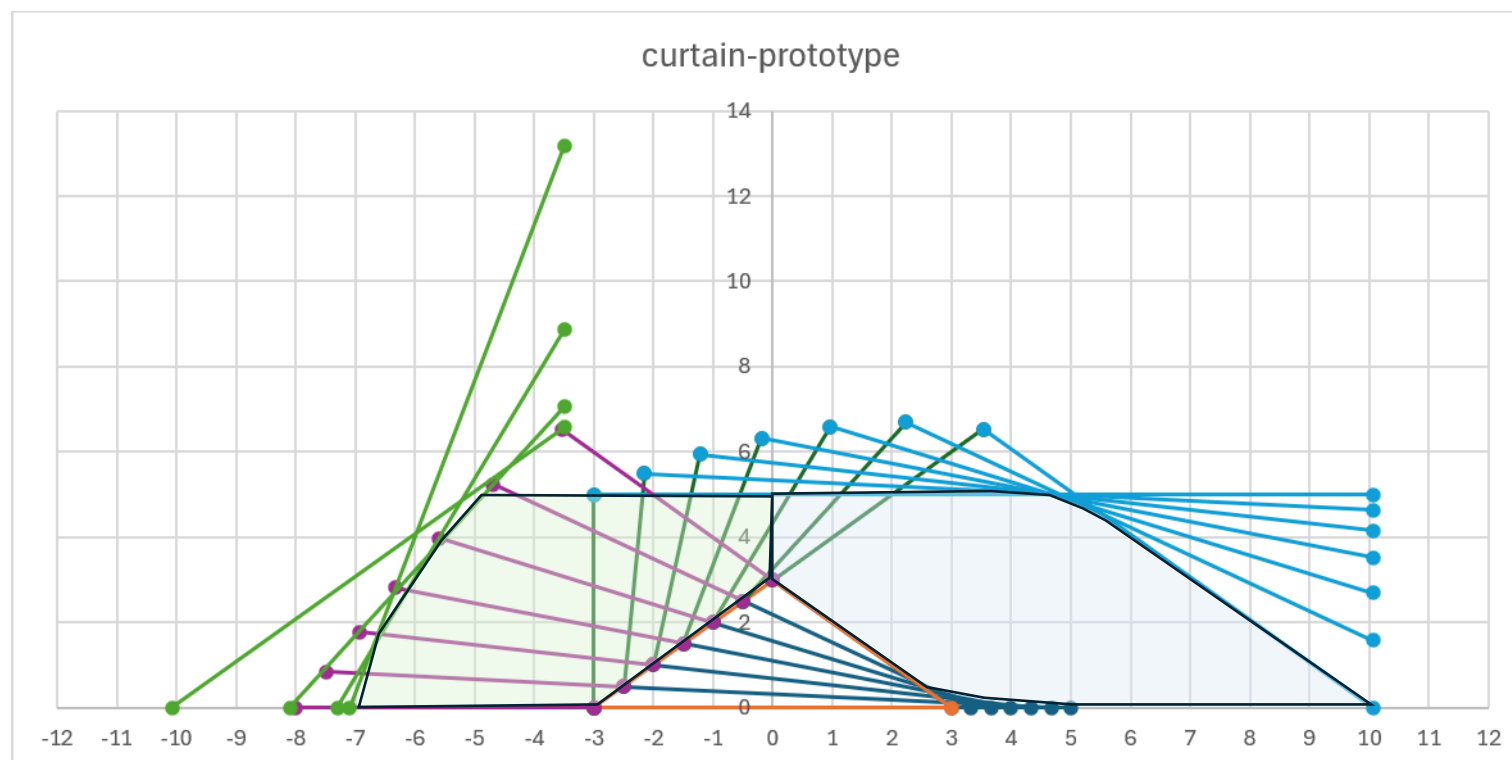
Where did that form come from?

The above form is derived using a continuously changing gap overlap, in such a way that the inside corner of the corridor follows the 45° lines of the maximal solution for the 45° middle position of any figure. The gap difference between left and right movement is 0 at that point, and then let it increase but while continuously staying within the triangle limits.

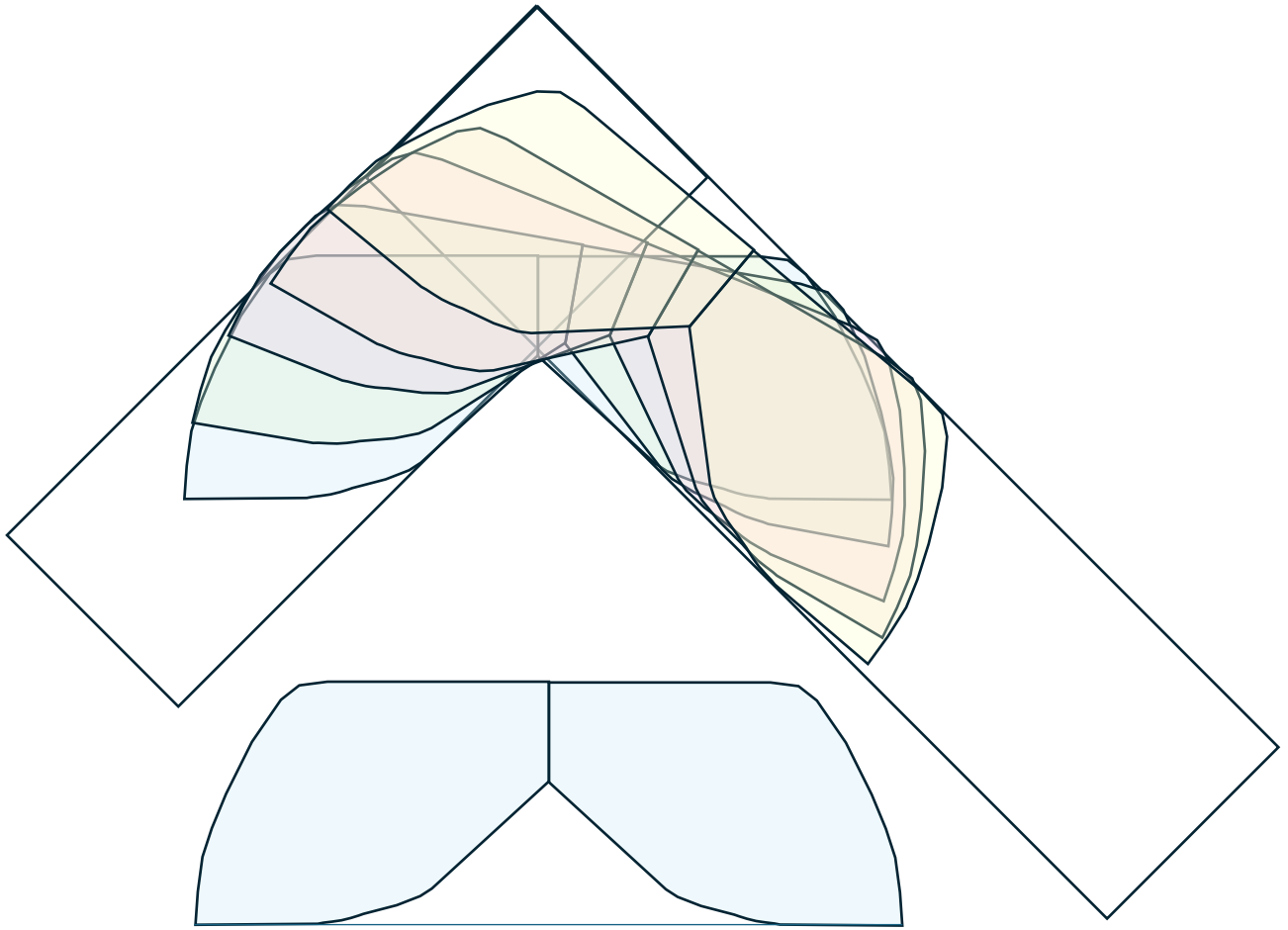
On the right side one can see the extended cut-out that happens at the bottom.

To complete the form the cuts driven by the corridor provide the external border.

The following figure shows part of the construction on the left half, the right half is symmetrical, and vice-versa.



Moving the “curtain through the corridor.



The picture is a rough creation from the computed excel pictures. It is slightly imprecise.

The solution contains a hidden parameter: the bottom curve size. It was considered not very relevant for this try-out to look for “best” size.

It provides a clear insight in the changes required to go from the Hammersley telephone to the better solution. It comes closer to a typical two-seater sofa too

.

Conclusion

In the end I do not know whether the optimal solution was found by Gerber, or if the proof it is optimal is correct. I did find a way to prove it by brute force. That method does provide solutions very close to the Gerber form by tweaking the parameters. The maximal area could be computed to the precision required, given enough computer resources to compute the intersections and areas of the results.

What I like very much is that the solution is clearly justifiable, no tricks required. While I spend a lot of time on it (some might say: too much) the results are there.

At this point I started wondering about other corner angles than 90° . It would be nice to find the 90° solution as just one of the possible corners. It worked.

References

The main point is that my approach started from a simple assumption, and provides a space that must contain the optimal solution. It is a matter of computing time to find the best too any precision required.

For other angles than 90° , I chose to limit myself to Hammersley-style sofa's, likely not optimal, but good approximations.

L. Moser. Problem 66-11: Moving furniture through a hallway. SIAM Rev. 8 (1966), 381.

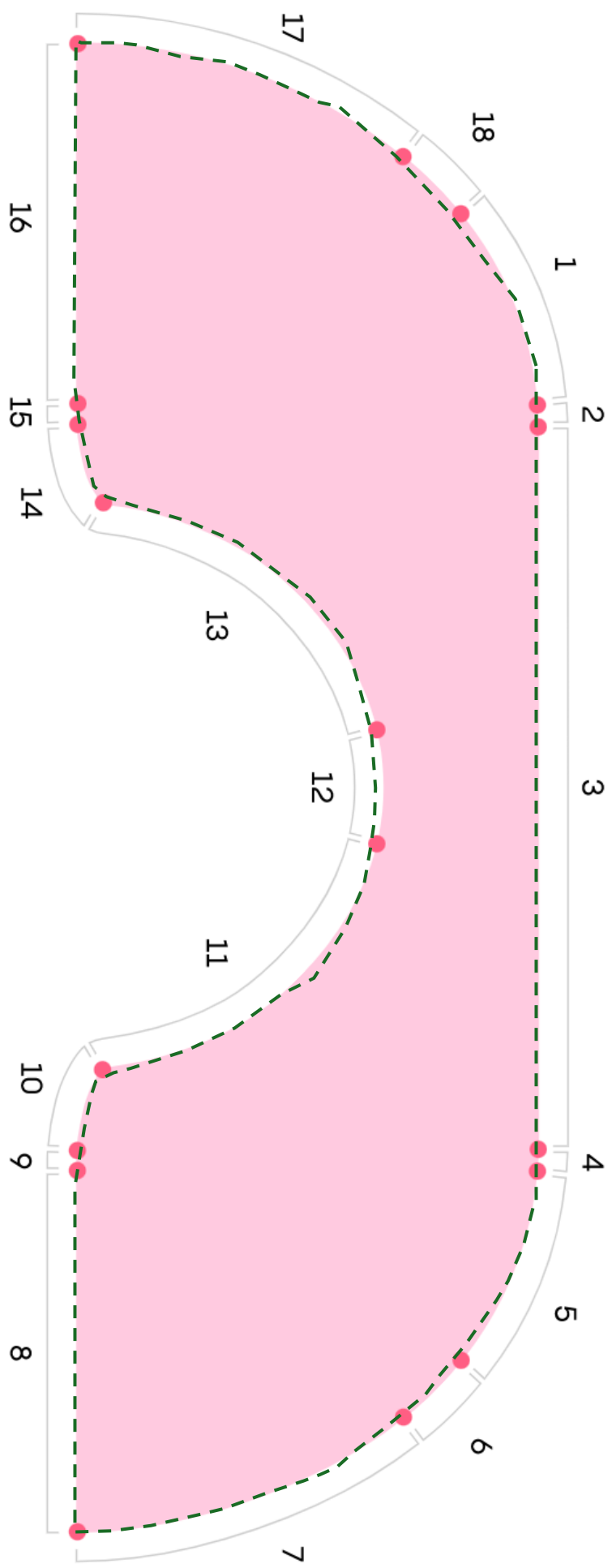
J. L. Gerver. On moving a sofa around a corner. Geom. Dedicata 42 (1992), 267–283.

J. M. Hammersley. On the enfeeblement of mathematical skills by “Modern Mathematics” and by similar soft intellectual trash in schools and universities. Bull. Inst. Math. Appl. 4 (1968), 66–85.

Improved upper bounds in the moving sofa problem, <https://arxiv.org/pdf/1706.06630>

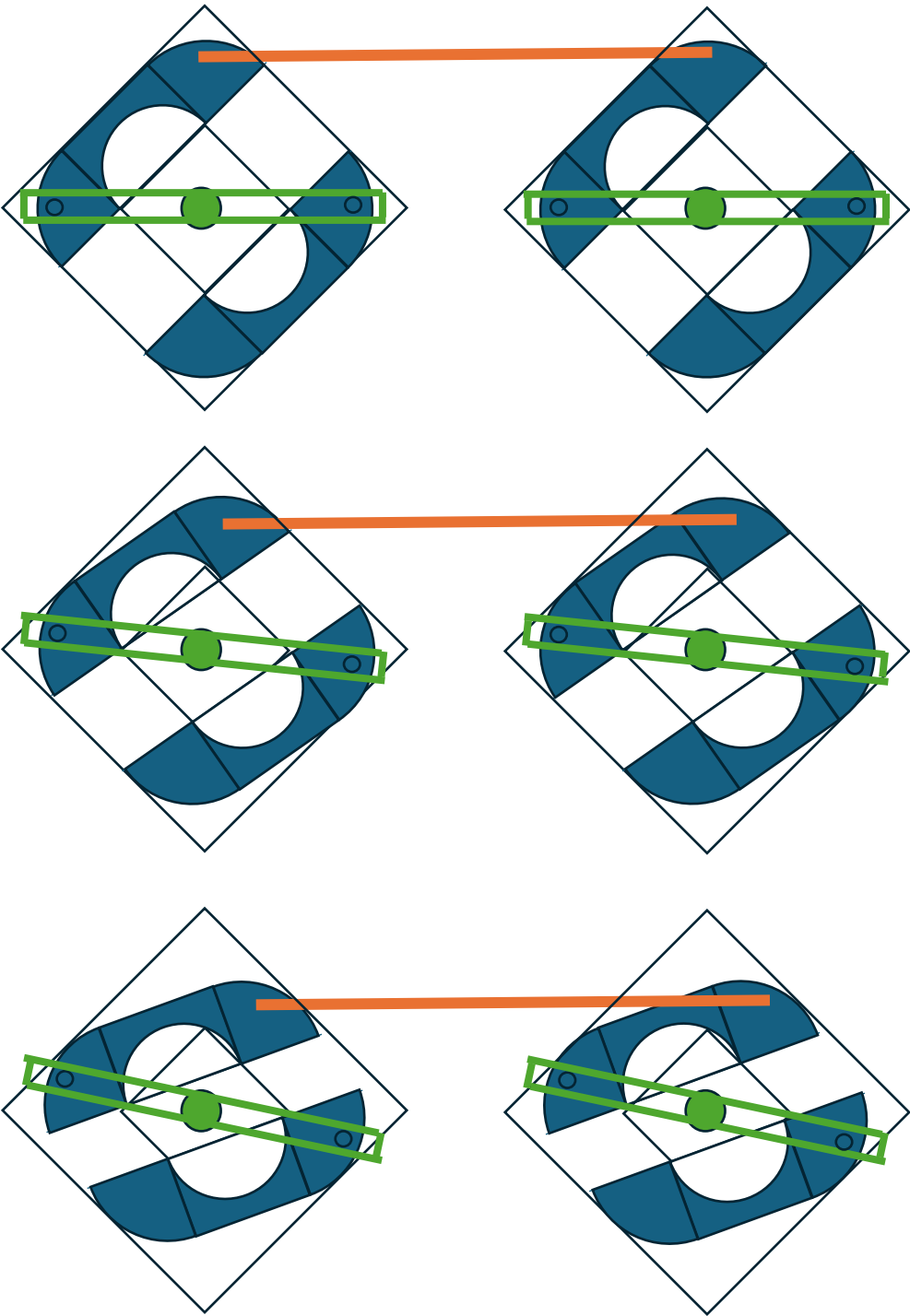
Optimality of Gerver's Sofa Jineon Baek1 December 2, 2024, <https://arxiv.org/pdf/2411.19826>

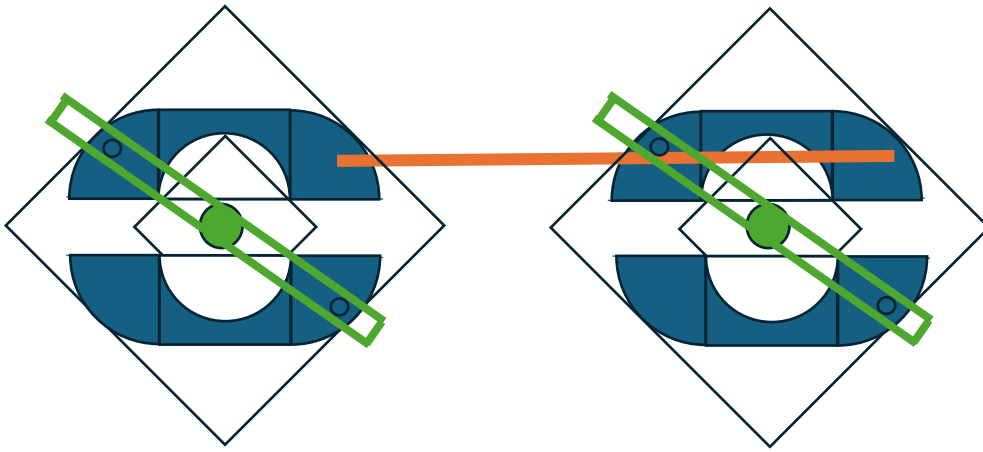
Appendix: compare a hand-made (not automated) solution with the “official” final solution



Appendix: a curious toy?

Note: these “toy” designs are copyrighted, 2026

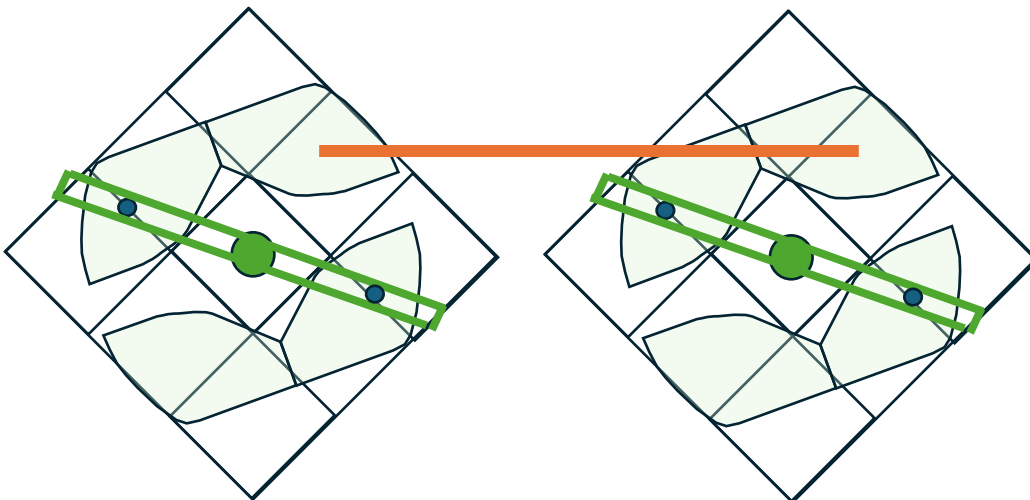
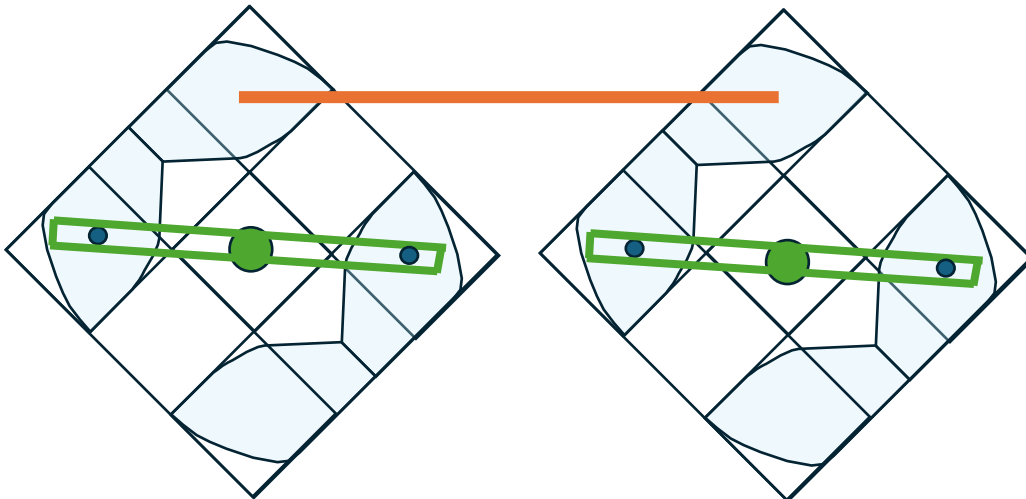


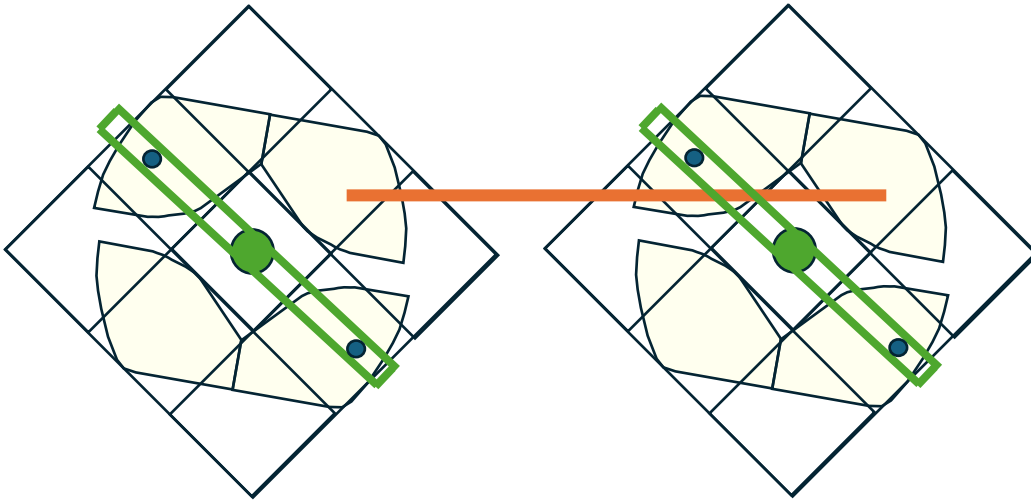


If the central axis of one of the figure turns

- the sofas will turn, the bar will move
- the sofas in the other one will turn,
- the central axis of the other will turn in sync with the driving axis
- Almost all of the time, there are only six contact points between the 2 sofas and the square house

You can do the same with the curtain:





This form would likely produce a smoother turn. For the phone-style one could have superimposed pairs with 90° shift to make the overall movement more balanced.

Another thing could be to add a second green connector or another red connector for more balanced forces.

It is also possible to extend the number of “wheels” to 3-4-6-8-9, ...

The 45° choice is also random, for dense packing, use 0° and they can be put side-by-side.

If the form would be air-tight, it will not turn easily: the air in the corners would create high pressure zones, or vacuum zones, or both.

On the other hand, the system can be cooled/oiled easily.

With some valves added in the corners it may be a pump.